Exercise 26

Use implicit differentiation to find an equation of the tangent line to the curve at the given point.

$$\sin(x+y) = 2x - 2y, \quad (\pi, \pi)$$

Solution

The aim is to evaluate y' at $x = \pi$ and $y = \pi$ in order to find the slope there. Differentiate both sides of the given equation with respect to x.

$$\frac{d}{dx}[\sin(x+y)] = \frac{d}{dx}(2x - 2y)$$

$$\left[\cos(x+y) \cdot \frac{d}{dx}(x+y)\right] = \frac{d}{dx}(2x) - \frac{d}{dx}(2y)$$

$$\left[\cos(x+y) \cdot (1 + y')\right] = (2) - (2y')$$

$$\cos(x+y) + y' \cos(x+y) = 2 - 2y'$$

Solve for y'.

$$[\cos(x+y) + 2]y' = 2 - \cos(x+y)$$

$$y' = \frac{2 - \cos(x+y)}{\cos(x+y) + 2}$$

Evaluate y' at $x = \pi$ and $y = \pi$.

$$y'(\pi, \pi) = \frac{2 - \cos(\pi + \pi)}{\cos(\pi + \pi) + 2} = \frac{1}{3}$$

Therefore, the equation of the tangent line to the curve represented by $\sin(x+y) = 2x - 2y$ at (π, π) is

$$y - \pi = \frac{1}{3}(x - \pi).$$

Below is a graph of the curve and the tangent line at (π, π).